MISSING MATH SKILLS: Students Just Don't Understand

By Leonard Geddes

Systems are designed to produce predictable results. In fact, expecting different results from the same system is a recipe for insanity. Thus, we can assume that any consistently produced outcomes are functions of the system.

What if the outcomes are bad? What does that say about the system? The relationship between professors and students is an academic system in which their interactions produce learning outcomes. What if numerous students consistently fail the class? The fault may not rest entirely on the students or on the professors. The system of interactions between them may be partially to blame.

If we want to change systemic learning outcomes, we must first understand the system. I propose that many math-based courses are operating with a major design flaw. The default modus operandi activates and rewards students' memorization skills at the expense of developing true understanding.

By understanding, I mean the ability to explain core concepts with accuracy and clarity. Without this fundamental level of knowledge, students cannot properly analyze or evaluate complex mathematical problems. I proposed a new system based on the ThinkWell-LearnWell™ Diagram (see page 2) that activates and rewards understanding, the core subskill students must possess before they can analyze and solve problems on tests and in the real world.

The Current Learning System

I have noted the following exchanges within the array of math courses I've observed:

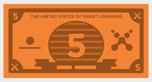
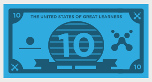
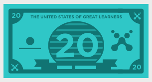
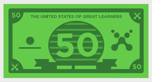
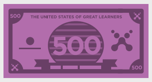
- Students expect to be assessed on "what" knowledge. This is baseline knowledge about the topics of the course; therefore, they focus on this level of cognitive work, which is level one (remembering) on the ThinkWell-LearnWell Diagram. Each time they produce knowledge using mental skills consistent with this mode of thinking, they trigger a confirmation signal that makes them feel good. However, they don't realize that professors expect students to already possess this level of knowledge; therefore, they do not assess this level of cognitive work.

- Students want professors to work through problems in class because solving problems is the most visible work of the course. Solving problems is deceptive (see more in *Student Success in STEM Fields*) to students and educators because it mimics the applying level (level three) of the diagram. However, unless students are transferring known concepts (level two, understanding) to problems they haven't previously seen, they are simply doing more memorization work. So while professors may be using their analyzing, evaluating, and applying skills while working through problems, much of the class may still be stuck using their memorization skills.
- Furthermore, when students work through problems away from class, they continue to ignore and devalue conceptual knowledge. They aren't interested in and simply don't know how to marshal the proper mental resources to develop this essential intelligence. Moreover, other than asking students whether they understand throughout their instruction, faculty aren't necessarily skilled at assisting students

ThinkWell-LearnWell™ Diagram

Lower-Order Thinking Skills → Higher-Order Thinking Skills

Surface Approach to Learning → Deep Approach to Learning

Metacognitive Learning Goals	Bloom's Higher-Order Thinking Skills	Corresponding Learning Outcomes	Outcome Valuation
To Identify or Define Information Students seek to answer some form of this what-based question: can I list and/or define the key terms?	**Remembering** Students work to recall/recognize information, ideas, and principles in the approximate form in which they were learned.	**Able to Recall or Duplicate Information** Students will be able to reproduce information in similar form as the original source. Corresponds to tasks in which cues are embedded.	
To Explain Information Students seek to answer some form of this why-based question: can I explain the reasoning behind the ideas/concepts?	**Understanding** Students work to explain and provide rationales to support concepts and/or principles.	**Able to Provide Rationales for Information** Students will be able to explain why concepts are essential to understanding the topic, subject, story, etc. Corresponds to tasks that require explanations or elaborations.	
To Apply Information to New Situations Students seek to answer some form of this how-based question: can I apply this information to a new or different situation, problem, or context?	**Applying** Students work to transfer principles and/or concepts to a different problem or task with minimal cues or direction.	**Able to Apply Information to Different Situations** Students will be able to use information to complete a problem or task with minimal direction or cues. Corresponds to tasks that require application of knowledge to a situation.	
To Compare and Contrast Information Student's seek to answer some form of this analytical question: can I distinguish processes, procedures, or principles from seemingly identical processes, procedures, or principles?	**Analyzing** Demands that students be able to distinguish and differentiate between comparable processes, functions, methods, etc.	**Able to Discern Nuances of Information** Students will be able to discern patterns, differences and similarities within information. Corresponds to tasks that require students to distinguish between similar sets of information, processes, or outcomes.	
To Make Judgments About Information Student's seek to answer some form of this evaluative question: can I determine the best rationale, plan, solution, course of action, etc., given the information?	**Evaluating** Demands that students be able to make judgments with information.	**Able to Reach Conclusions with Information** Students will be able to make judgments about information they've analyzed. Corresponds to tasks that require students to decide which course of action, solution, or option is best.	
To Introduce, Develop a Viewpoint Students seek to answer some form of this generative question: can I synthesize the information in an original way?	**Creating** Demands that students be able to construct new information from existing information.	**Able to Produce New Information** Students will be able to present new meaning or generate new knowledge. Corresponds to tasks that require students to produce authentic work.	

in building their understanding. They are, after all, subject-matter experts, not necessarily teaching experts. So we should not be surprised when students lack the understanding needed for their assessments.

The net effect of the system is that students spend most of their time trying to cram as many rules, formulas, and methods as possible into their minds. They hope to effectively match the correct rules, formulas, and methods to problems on tests and tasks. But when they face problems that they haven't previously seen and in unfamiliar contexts, they realize that they're unable to solve such problems.

This experience may cause them to feel unfairly treated because the tests require them to solve problems they have not yet encountered. As a result, they want their professors to devote even more class time to solving problems, which pressures faculty to do more problem-solving. The cycle reinforces and repeats itself.

This system works only if students are expected to solve problems exactly like those presented in class. If assessments have relatively few problems—enough that students can rely on their recollections—and the problems are nearly identical to those students have previously seen, then students can focus their academic work on trying to memorize and retrieve all the various ways they've seen problems solved. But the current system does not operate this way.

The Design Flaw in the Current System

Systems typically fail to produce the desired outcomes because they either have an inherent design flaw or because the system components are not properly maintained. The math-teaching system has a fundamental design flaw. To understand this flaw, we must first comprehend what complex problem-solving entails.

> **Any consistently produced outcomes are functions of the system.**

Complex problem-solving is a highly analytical and evaluative process. It requires students to use their knowledge and cognitive skills to solve unpredictable problems that manifest in unknowable contexts. By "solving unpredictable problems," I mean that students have not previously seen the problems; therefore, they cannot rely upon recollections of specific problems they've encountered in the past to predict what problems will appear on future assessments. By "manifest in unknowable contexts," I mean that the specific details and conditions of the problems are infinite, so they can't be known.

To solve complex problems, students must possess the requisite conceptual knowledge that enables them to analyze the problem. Only after such analysis can they select the proper method, rule, or formula required to solve the problem.

For example, before students can begin solving a complex problem, they must first analyze the problem's context. This initial process requires them to consider all the relevant information regarding the problem, including explicitly expressed information and any implied elements. Then they must decide which concept(s), rule(s), or method(s) are appropriate, given the specific context of the problem. This type of cognitive work is represented by levels four and five of the

> **Students do not sustain this system alone. Professors reinforce the flawed system.**

diagram. Keep in mind that this activity occurs before students can solve the problem, which is an applying skill, represented by level three of the diagram.

While a few outliers may sneak through, a flawed system cannot consistently produce students who can generate what I deduce is the macro-outcome of most courses: using their knowledge and cognitive skills to solve unpredictable problems that manifest in unknowable contexts. Instead, it will produce students who believe they can learn to solve problems by watching professors work through problems or by solving problems vicariously via online resources. This type of assumed learning is dangerous because it can deceive students (and educators) into believing they can do things they are unable to do.

Students do not sustain this system alone. Professors reinforce the flawed system. By working through problems in class without explicitly and strategically covering key concepts, they unknowingly sustain the system. Hence, the environment is producing very predictable results, regardless of the inadequacy of the outcomes.

A New System for Producing Better Learning Outcomes

During a recent visit to a college campus, I observed math faculty working through problems with their classes. They got through about three problems per class. Professors and a few student thought leaders did nearly 90 percent of the cognitive lifting for the class, while most students "learned" vicariously through others' efforts, rather than actively doing their own cognitive work.

This system satisfies students and professors, but it produces an unhealthy division of labor by overburdening faculty and underworking students. As a result, when students encounter cognitive challenges, they look outwardly to their professors for help rather than looking within their own knowledge base and abilities. In the current system, no amount of studying will prepare students for the types of cognitive work they must be able to do. By making modest changes to the existing system, however, professors can position students to produce satisfactory outcomes.

A new system will produce different results. If professors are to implement the new system, however, they must perceive that the benefits significantly outweigh the costs of change. Since professors are currently working through approximately three problems in a fifty-minute class, I created a similar structure with some strategic changes. I call it the 3x10 structure. The objective is to cover three problems in class with a different distribution of labor.

The professor divides class into five ten-minute blocks:

1. The professor covers the "why" of key concepts. This is level two of the ThinkWell-LearnWell™ Diagram.
2. The professor works through problem 1 while students observe and participate.
3. Students divide into groups to work through problem 2.
4. Students work individually on problem 3.
5. The professor reviews the key concepts and answers questions.

THE 3X10 STRUCTURE

In the current system, depicted to the right, the labor rests heavily on the professor. With the new 3x10 Structure below, labor is divided to prepare students to solve problems on their own.

Problem 1
Professor works through the problem.

Problem 2
Professor works through the problem.

Problem 3
Professor works through the problem.

10 MINUTES

Problem 1
Professor works through the problem.

10 MINUTES

Problem 2
Students work through the problem in groups.

10 MINUTES

Problem 3
Students work through the problem individually.

Students watch their professor work through problems.

Professor elicits feedback and further instructs.

Students solve in the safety of a group.

Professor captures group feedback and uses it to further instruct.

Students are better positioned to work independently.

Students get a better sense of what they can and cannot do independently.

The 3x10 structure allows faculty to model problem-solving. Then it allows students to attempt to work through problems within the safety and security of the group. Finally, students will be able to test their individual knowledge and abilities, thus gaining metacognitive awareness of what they can and cannot do. This system shifts the cognitive load from faculty and onto students while providing structure and support for them to be successful.

In one of the calculus classes I observed, here is how the new system would have applied. The lesson outcome for level two of the ThinkWell-LearnWell Diagram would have been "explain why the physics concepts of position, velocity, and acceleration are important within the context of calculus." Expressing these relationships in this manner would trigger students to transfer the concepts that they likely learned in physics to the current calculus context. The students could only perform this mental task, if they understood the three concepts (position, velocity, and acceleration). This is level two cognitive labor.

> **The 3x10 structure shifts the cognitive load from faculty and onto students while providing structure and support for them to be successful.**

If students could not perform this task, then it would have been a metacognitive indicator that they were missing the requisite knowledge needed to perform it. So whether students could or could not transfer the three concepts from the physics context to the calculus context was not a matter of background knowledge (a common misconception). It was a difference in the quality of mental work they had performed when learning the material. Students who accurately recalled the concepts in physics but did not understand the concepts in their current calculus context would have failed test questions on the material. Their failure would not stem from a lack of study or lack of knowledge; rather, it would be due to a lack of sufficient processing with the proper cognitive skills.

However, if the professor had metacognitively prompted the students by getting them to explicitly specify and explain the concepts before trying to work through problems, he would have evoked deep cognitive processing among students resembling the progression below:

- Students would have segregated each of these elements in their minds, checking to see if they knew what each concept was in isolation. Depending on the internal feedback they generated, they would have either known they needed to review these concepts or progress their learning. This activity is performed on level one of the diagram.
- Leveraging the foreknowledge of the learning outcome, students would have pondered how the concepts translated to their current calculus material. This critical cognitive leap would have enabled them to again assess their knowledge. This time, however, they would not be working on level one, trying to assess their degree of recall. They would be assessing how the concepts fit within the context of calculus. *Being able to contextualize each of the concepts within the context of calculus is the exact type of deeper understanding that was previously avoided.* This activity progresses the students to level two of the diagram.
- This work immediately benefits students throughout the course by putting them in stronger positions to work along with the professor (extracting more meaningful

learning from the professor's work), with their peers (each contributing more to the work), and individually (more confidently and competently through inevitable challenges).

Metacognitive Bonus

Most educators would be satisfied with the old system of interaction. By explicitly using the ThinkWell-LearnWell Diagram along with the 3x10 instruction, however, the professor would have installed a metacognitive framework within students' minds.

In a manner of weeks, the students will have added an essential element of metacognitive infrastructure to their academic work toolbox, and they will use these tools in every future course they take. The students won't just be learning content; they will become more productive learners.

The key is to constantly ask what type of cognitive action the work requires. As you make this self-questioning explicit to your class, students will internalize this key metacognitive regulatory question and will become more competent learners as a result. Now, this system will produce the predictable results that you desire and that your students need.

Leonard Geddes is founder of The LearnWell Projects, an academic success organization devoted to making learning more visible, manageable, and effective. His work leverages metacognition research to optimize student learning, enhance faculty instruction, and improve institutional outcomes. Leonard provides workshops and trainings for colleges and universities throughout North America.

Testimonials

"The 3X10 structure actually helped me teach less but get better results!"

(ANONYMOUS PROFESSOR SURVEY RESPONSE)

"I always assumed that completing homework assignments was preparing me for exams. Now I know how to check the skills I am using to make sure they match my professors' outcomes."

(JUNIOR, UNIVERSITY OF CALIFORNIA AT SAN DIEGO)

"My lower-performing math students now have some of the highest grades in my classes, thanks to this process. It just feels good!"

(INSTRUCTOR, CALDWELL COMMUNITY COLLEGE)

Useful Links

PME Regulation Process
https://youtu.be/V273Zqb4mSM

Differentiating Students' Thinking Skills pt. 1
https://youtu.be/cuB_4jImrZk

Differentiating Students' Thinking Skills pt. 2
https://youtu.be/eaTGkBLGsu0

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